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Soft Sub Spaces and Soft b-Separation Axioms in Binary Soft Topological Spaces

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Abstract – In this article, we introduce binary soft pre-separation axioms in binary soft topological space along with several properties of binary soft pre$\tau_{\triangle_i}$, i = 0; 1; 2, binary soft pre regular, binary soft pre$\tau_{\triangle_3}$, binary soft pre normal and binary soft $\tau_{\triangle_4}$ axiom using binary soft points. We also mention some binary soft invariance properties namely binary soft topological property and binary soft hereditary property. We hope that these results will be useful for the future study on binary soft topology to carry out general background for the practical applications and to solve the thorny problems containing doubts in different grounds.

***Keywords* –** *Binary soft topology, binary soft pre-open sets, binary soft pre closed sets, binary soft pre separation axioms.*

1 Introduction

The concept of soft sets was first introduced by Molodtsov [3] in 1999 as a general mathematical technique for dealing with uncertain substances. In [3,4] Molodtsov magnificently applied the soft theory in numerous ways, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. Point soft set topology deals with a non-empty set X

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together with a collection τ of sub set X under some set of parameters satisfying certain conditions. Such a collection τ is called a soft topological structure on X. General soft topology studied the characteristics of sub set of Xby using the members of. Therefore the study of point soft topology can be thought of the study of information. But in the real world situation there may be two or more universal sets. Our attempt is to introduce a single structure which carries the sub sets of X and Y for studying the information about ordered pair of sub sets of X and Y. Such a structure is called a binary soft structure from X to Y.

In 2016 Açıkgöz andTas [1] introduced the notion of binary soft set theory on two master sets and studied some basic characteristics. In prolongation, Benchalli et al. [2] planned the idea of binary soft topology and linked fundamental properties which are defined over two master sets with appropriate parameters. Benchalli et al. [6] threw his detailed discussion on Binary Soft Topological. Kalaichelvi and Malini [7] beautifully discussed Application of Fuzzy Soft Sets to Investment Decision and also discussed some more results related to this particular field. Özgür and Taş, [8] studied some more Application of Fuzzy Soft Sets to Investment Decision Making Problem. Taş et al. [9] worked over An Application of Soft Set and Fuzzy Soft Set Theories to Stock Management Alcantud et al. [10] carefully discussed Valuation Fuzzy Soft Sets: A Flexible Fuzzy Soft Set Based Decision Making Procedure for the Valuation of Assets. Çağman and Enginoğlu [11] attractively explored Soft Matrix Theory and some very basic results related to it and its Decision Making.

In continuation, in the present paper we have defined and explored several properties of binary soft b-$\tau_{\triangle_i}, i = 0; 1; 2$ binary soft b-regular, binary soft b-$\tau_{\triangle_3}$, binary soft b-normal and binary soft b-$\tau_{\triangle_4}$ axioms using binary soft points. Also, we have talked over some binary soft invariance properties i.e. binary soft topological property and binary soft hereditary property in binary soft topological spaces.

The arrangement of this paper is as follows: Section 1 briefly reviews some basic concepts about soft sets, binary soft sets and their related properties; Section 2 some hereditary properties are discussed in a beautiful way. Section 3 is devoted to Binary Soft b-Separation Axioms. Section 4 is devoted to Binary Soft b-Regular, Binary Soft b-Normal and Binary b-Soft $\boldsymbol{\tau}_{\Delta_i}$ (i=4, 3) Spaces.

2. Preliminaries

Definition 2.1. [5] Let X be an initial universe and let E be a set of parameters. Let P(X) denote the power set of X and let A be a nonempty subset ofE. A pair (F, A) iscalled a soft set over X, where F is a mapping given by $F: A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\in A$,F (ε) may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Let U_1, U_2be two initial universe sets and E be a set of parameters.

Let$P(U_1)$, $P(U_2)$ denote the power set of U_1, U_2 respectively. Also, let $A, B, C \subseteq E$.

Definition 2.2. [1] A pair (F, A) is said to be a binary soft set over $U_1, U_2$where F is defined as below:

$F: A \rightarrow P(U_1) \times P(U_2)$, $F(e) = (X, Y)$ for each $e \in A$ such that $X \subseteq U_1$, $Y \subseteq U_2$.

Definition 2.3. [1] A binary soft set (G, A) over U_1, U_2 is called a binary absolute soft set, denoted by$\tilde{\tilde{A}}$if F (e) =(U_1, U_2) for each$e \in$ A.

Definition 2.4. [1] The intersection of two binary soft sets of (F, A) and (G, B) over the common U_1, U_2 is the binary soft set (H, C), where $C = A \cap B$ and for all $e \in C$

$$H(e) = \begin{cases} (X_1, Y_1) \text{ if } e \in A - B \\ (X_2, Y_2) \text{ if } e \in B - A \\ (X_1 \cup X_2, Y_1 \cup Y_2) \text{ if } e \in A \cap B \end{cases}$$

Such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$ for each$e \in$ B. We denote it $(F, A) \tilde{\tilde{\cup}} (G, A) = (H, C)$

Definition 2.5. [1] The intersection of two binary soft sets (F, A) and (G, B) over a common U_1, U_2 is the binary soft set (H, C), where

$$C = A \cap B, \text{ and } H(e) = (X_1 \cap X_2, Y_1 \cap Y_2)$$

for each $e \in C$ such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$ for each $e \in B$. We denote it as $(F, A) \tilde{\tilde{\cap}} (G, B) = (H, C)$

Definition 2.6. [1] Let (F, A) and (G, B) be two binary soft sets over a commonU_1, U_2. (F, A) is called a binary soft subset of (G, B) if

(i) $A \subseteq B$,
(ii) $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ Such that $F(e) = (X_1, Y_1)$, $G(e) = X_2, Y_2$ for each$e \in$ A.

We denote it as $(F, A) \tilde{\tilde{\subseteq}} (G, B)$.

Definition 2.7. [1] A binary soft set (F, A) over U_1, U_2 is called a binary null soft set, denoted by if $F(e) = (\varphi, \varphi)$ for each $e \in A$.

Definition 2.8. [1] The difference of two binary soft sets (F, A) and (G, A) over the Common U_1, U_2 is the binary soft set (H, A), where $H(e)$ $(X_1 - X_2, Y_1 - Y_2)$ for each $e \in A$ such that $(F, A) = (X_1, Y_1)$ and $(G, A) = (X_2, Y_2)$.

Definition 2.9. [2] Let $\tau_\triangle$ be the collection of binary soft sets over U_1, U_2 then $\tau_\triangle$ issaid to be a binary soft topology on U_1, U_2 if

(i) $\tilde{\tilde{\varphi}}, \tilde{\tilde{X}} \in \tau_\triangle$
(ii) The union of any member of binary soft sets in $\tau_\triangle$ belongs to $\tau_\triangle$
(iii) The intersection of any two binary soft sets in $\tau_\triangle$ belongs to$\tau_\triangle$

Then $(U_1, U_2, \tau_\triangle, E)$ is called a binary soft topological space over U_1, U_2.

Definition 2.10. [2] Let $(U_1, U_2, \tau_\triangle, E)$ be a binary soft topological spaces on $\tilde{X}$ over $U_1 \times U_2$ and $\tilde{Y}$ be non empty binary soft subset of $\tilde{\tilde{X}}$. Then $\tau_{\triangle_Y} = \{^Y (F, E)/(F, E) \in \Delta$ is said to be the binary soft relative topology on $\tilde{\tilde{Y}}$and $(\tilde{\tilde{Y}}, \tau_{\triangle_Y}, E)$ is called a binary soft subspace of $(U_1, U_2, \tau_\Delta, E)$. We can easily verify that $\tau_{\triangle_Y}$ is a binary soft topology on $\tilde{\tilde{Y}}$.

Example 2.1. [2] Any binary soft subspace of a binary soft indiscrete topological space is binary soft indiscrete topological space.

Definition 2.11. Let (F, A) be any binary soft sub set of a binary soft topological space $(U_1, U_2, \tau_\triangle, E)$then (F, A) is called

1) Binary soft b-open set of $(U_1, U_2, \tau_\triangle, E)$ if$(F, A) \subseteq cl(int((F, A) \cup in(cl((F, A)$
2) Binary soft b-closed set of $(U_1, U_2, \tau_\triangle, E)$ if$(F, A) \supseteq cl(int(F, A)))in(cl(F, A)))$)

The set of all binary *b-open soft* sets is denoted by BSBO(U) and the set of all binary b-closed sets is denoted by BSBO(U).

Proposition 2.1. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological spaces on $\tilde{X}$ over $U_1 \times U_2$ and $\tilde{Y}$ be a non-empty binary soft subset of $\tilde{\tilde{X}}$. Then $(U_1, U_2, \tau_{\triangle_Y}, \alpha)$ is subspace of $(U_1, U_2, \tau_{\triangle_Y}, E)$ for each $\alpha \tilde{\tilde{\in}} E$.

Proof. Let $(U_1, U_2, \tau_{\triangle_Y}, \alpha)$ is a binary soft topological space for each$\alpha \in E$. Now by definition for any $\alpha \in E$

$$\begin{aligned} \tau_{\triangle_Y} &= \{^Y F(\alpha)/(F, E) \text{is binary soft b} - \text{open set}\} \\ &= \{\tilde{\tilde{Y}} \cap F(\alpha)/(F, E) \text{ isbinary soft b} - \text{open set}\} \\ &= \{\tilde{\tilde{Y}} \cap F(\alpha)/(F, E)\text{isbinary soft b} - \text{open set} \\ &= \{\tilde{\tilde{Y}} \cap F(\alpha)/F(\alpha) \in \tau_{\triangle_\alpha}\} \end{aligned}$$

Thus $(U_1, U_2, \tau_{\triangle_Y}, \alpha)$ is a subspace of$(U_1, U_2, \tau_\Delta, \alpha)$.

Proposition 2.2. Let $(U_1, U_2, \tau_{\triangle_Y}, E)$ be a binary soft subspace of a binary softTopological space $(U_1, U_2, \tau_\Delta, E)$ and (G, E) be a binary soft b-open in $\tilde{Y}$. If $\tilde{\tilde{Y}} \in \tau_\Delta$, Then (G, E) $\in \tau_\Delta$.

Proof. Let (G, E) be a binary soft b-open set in $\tilde{\tilde{Y}}$, then there exists a binary soft b-open set (H, E) in $\tilde{\tilde{X}}$ over $U_1 \times U_2$ such that (G,E) = $\tilde{\tilde{Y}} \cap (H, E)$. Now, if $\tilde{\tilde{Y}} \in \tau_\Delta$, then $\tilde{\tilde{Y}} \cap (H, E) \in \tau_\Delta$ by the third axiom of the definition of binary soft topological space and hence (G, E) $\in \tau_\Delta$.

Proposition 2.3. Let $(U_1, U_2, \tau_{\triangle_Y}, E)$ be a binary soft subspace of a binary soft topological space $(U_1, U_2, \tau_\Delta, E)$ and (G, E) be a binary soft b-open set of $\tilde{\tilde{X}}$ over $U_1 \times U_2$, then

(i) (G, E) is binary soft b-open in $\tilde{\tilde{Y}}$ if and only if $(G, E) = \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (H, E)$ for some $(H, E) \in \tau_\Delta$.
(ii) (G, E) is binary soft b-closed in $\tilde{\tilde{Y}}$ if and only if $(G, E) = \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (H, E)$ for some binary soft b-closed set in $(H, E) \in \tilde{\tilde{X}}$ over $U_1 \times U_2$.

Proof. (i) Follows from the definition of binary soft subspace.

(ii) If (G, E) is binary soft b-closed in $\tilde{\tilde{\tilde{Y}}}$ then we have $(G,E) = \tilde{\tilde{\tilde{Y}}}$, then we have $(G,E) = \tilde{\tilde{\tilde{Y}}} - (H,E)$, for some binary soft b-open $(H,E) \in \tau_{\Delta_Y}$, now $(H,E) = \tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} (H,E)$ for some binary soft b-open $(K,E) \in \tau_\Delta$ for any $\beta \in E$,

$$
\begin{aligned}
G(\beta) &= \tilde{\tilde{\tilde{Y}}}(\beta) - H(\beta) \\
&= \tilde{\tilde{\tilde{Y}}} - H(\beta) \\
&= \tilde{\tilde{\tilde{Y}}} - [\, \tilde{\tilde{\tilde{Y}}}(\beta) \cap K(\beta)] \\
&= \tilde{\tilde{\tilde{Y}}} - [\, \tilde{\tilde{\tilde{Y}}} \cap K(\beta)] \\
&= \tilde{\tilde{\tilde{Y}}} - K(\beta) \\
&= \tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} (\tilde{\tilde{\tilde{X}}} - K(\beta))] \\
&= \tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} [K(\beta)] \\
&= \tilde{\tilde{\tilde{Y}}}(\beta) \tilde{\tilde{\cap}} [K(\beta)]^C
\end{aligned}
$$

Thus $(G,E) = \tilde{\tilde{\tilde{Y}}}(\beta) \tilde{\tilde{\cap}} [K(\beta)]^C$ Where $(K,E)^C$ is binary soft b-closed set in $\tilde{\tilde{\tilde{X}}}$ over $U_1 \times U_2$ as $(K,E) \in \tau_\Delta$.

Conversely, assume that $(G,E) = \tilde{\tilde{\tilde{Y}}} \cap (H,E)$ for some binary soft b-closed set (H,E) in $\tilde{\tilde{\tilde{X}}}$ over $U_1 \times U_2$ which means that$(H,E) \in \tau_\Delta$. Now if $(H,E) = \tilde{\tilde{\tilde{X}}} - (K,E)$ where $(K,E) \tilde{\tilde{\in}} \tau_\Delta$ then for any $\beta \tilde{\tilde{\in}} E$,

$$
\begin{aligned}
G(\beta) &= \tilde{\tilde{\tilde{Y}}}(\beta) \tilde{\tilde{\cap}} H(\beta) \\
&= \tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} H(\beta) \\
&= \tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} [\tilde{\tilde{\tilde{X}}} - K(\beta)] \\
&= \tilde{\tilde{\tilde{Y}}} - [\tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} K(\beta)] \\
&= \tilde{\tilde{\tilde{Y}}}(\beta) - [\tilde{\tilde{\tilde{Y}}}(\beta) \tilde{\tilde{\cap}} K(\beta)] \\
&= \tilde{\tilde{\tilde{Y}}} - [\tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} (K,E)].
\end{aligned}
$$

Since$(K,E) \in \tau_\Delta$, so $[\tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} (K,E)] \in \tau_{\Delta_Y}$ and hence (G, E) is binary soft b- closed set in $\tilde{\tilde{\tilde{Y}}}$. This finishes the proof.

Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space. Let $(U_1, U_2, \tau_{\Delta_Y}, E)$ be a binary soft subspace of $(U_1, U_2, \tau_\Delta, E)$. Let $(F,E) \tilde{\tilde{\subseteq}} \tilde{\tilde{\tilde{Y}}}$ be a binary soft subset of $\tilde{\tilde{\tilde{Y}}}$. Thenwe can find the binary soft b-closure of (F, E) in the space $(U_1, U_2, \tau_{\Delta_Y}, E)$. The binary soft b-closure of (F, E) in $(U_1, U_2, \tau_{\Delta_Y}, E)$ is denoted by $\overline{\overline{(F,E)}}^{y}$.

Proposition 2.4. Let $(U_1, U_2, \tau_{\Delta_Y}, E)$ be a binary soft subspace of binary soft topological space $(U_1, U_2, \tau_\Delta, E)$. Let $(F,E) \tilde{\tilde{\subseteq}} \tilde{\tilde{\tilde{Y}}}$ be a binary soft subset of $\tilde{\tilde{\tilde{Y}}}$. Thenwe have the following results as follows.

(i) $\overline{\overline{(F,E)}}^{y} = \tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} \overline{\overline{(F,E)}}$.

(ii) $(F,E)^{*y} = \tilde{\tilde{\tilde{Y}}} \tilde{\tilde{\cap}} (F,E)^*$

(iii) $\underline{\underline{(F,E)}}_y \tilde{\tilde{\subseteq}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} \underline{\underline{(F,E)}}$

Proof. (i) To prove, let $\overline{\overline{(F,E)}}^y = \tilde{\tilde{Y}} \tilde{\tilde{\cap}} \overline{\overline{(F,E)}}$. We have $\overline{\overline{(F,E)}}^y$ =the binary soft intersection of all thebinary soft b-closed sets containing (F, E) = $\tilde{\tilde{\cap}}${(G, E) $_y$: (G, E) $_y$ is τ_{Δ_Y}-binary soft b-closed set and (G, E) $_y\tilde{\tilde{\supseteq}}$(F, E)} = $\tilde{\tilde{\cap}}${$\tilde{\tilde{Y}} \tilde{\tilde{\cap}}$ (G, E): (G, E) is -binary soft b-closed set and $\tilde{\tilde{Y}} \tilde{\tilde{\cap}}$ (G, E)$\tilde{\tilde{\supseteq}}$ (F, E)} = $\tilde{\tilde{\cap}}${$\tilde{\tilde{Y}} \tilde{\tilde{\cap}}$ (G, E): (G, E) is τ_{Δ_Y}-binary soft b-closed set and (G, E)$\tilde{\tilde{\supseteq}}$ (F, E)}= $\tilde{\tilde{Y}} \tilde{\tilde{\cap}}${$\tilde{\tilde{\cap}}$ (G, E): (G, E) is τ_Δ -binary soft b-closed set and (G, E)$\tilde{\tilde{\supseteq}}$ (F, E)} =$\tilde{\tilde{Y}} \tilde{\tilde{\cap}} \overline{\overline{(F,E)}}$. Thus $\overline{\overline{(F,E)}}^y = \tilde{\tilde{Y}} \tilde{\tilde{\cap}} \overline{\overline{(F,E)}}$.

(ii) To prove that (F, E) ^{y}= $\tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F,E)^*$,we know that, (F,E) The binary soft union of all the τ_{Δ_Y}-binary soft b-open Sets contained in (F,E)= $\tilde{\tilde{\cup}}$ {(H, E): (H, E) isτ_{Δ_Y}-binary soft b-open and (H, E)$\tilde{\tilde{\supseteq}}$ (F, E)}= $\tilde{\tilde{\cup}}$ {(H, E) =$\tilde{\tilde{Y}} \tilde{\tilde{\cap}}$ (K,E): (K, E) is τ_Δ-binary soft b-open set and $\tilde{\tilde{Y}} \tilde{\tilde{\cap}}$ (K,E) $\tilde{\tilde{\supseteq}}$ (F, E)}. Also we know that (F, E) ^{e} = $\tilde{\tilde{Y}} \tilde{\tilde{\cap}}$ [$\tilde{\tilde{\cup}}$(L, E) γ]: (L, E) is τ_Δ-binary soft b-open set and (L, E)γ $\tilde{\tilde{\subseteq}}$ (F, E)}. Now let $(M,E) \tilde{\tilde{\in}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F,E)^*$ which implies $(M,E) \tilde{\tilde{\in}} \tilde{\tilde{Y}}$ and $(M,E) \tilde{\tilde{\in}} (F,E)^*(M,E) \tilde{\tilde{\in}} \tilde{\tilde{Y}}$and $(M,E) \tilde{\tilde{\in}}\tilde{\tilde{\cup}}$(L,E) γ : (L,E) γ is τ_Δ -binary soft b-open set and $(L,E)\,\gamma \tilde{\tilde{\subseteq}} (F,E) \Rightarrow (M,E) \tilde{\tilde{\in}} \tilde{\tilde{Y}}and(M,E) \tilde{\tilde{\in}} (L,E)\,\gamma$, where $(L,E)_{\gamma_i}$, is τ_Δ -b-open and $(L,E)_{\gamma_i} \tilde{\tilde{\subseteq}} (F,E) \Rightarrow (M,E) \tilde{\tilde{\in}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (L,E)_{\gamma_i}$. Where $(L,E)_{\gamma_i}$ isτ_Δ-b-open and $(L,E)_{\gamma_i} \tilde{\tilde{\subseteq}} (F,E)$ that is $\tilde{\tilde{Y}} \tilde{\tilde{\cap}} (L,E)_{\gamma_i} \tilde{\tilde{\subseteq}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F,E) \Rightarrow (M,E) \tilde{\tilde{\in}}\tilde{\tilde{\cup}} \{\tilde{\tilde{Y}} \tilde{\tilde{\cap}} (K,E): (K,E)$ is τ_Δ- b-open $\tilde{\tilde{Y}} \tilde{\tilde{\cap}} (K,E) \tilde{\tilde{\subseteq}} (F,E)\} \Rightarrow (M,E) \tilde{\tilde{\in}} (F,E)^{*y}$. Thus $(M,E) \tilde{\tilde{\in}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F,E)^*$ which implies $(M,E) \tilde{\tilde{\in}} (F,E)^*$. Therefore $\tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F,E)^* \tilde{\tilde{\subseteq}} (F,E)^*$.

(iii) To prove, $\underline{\underline{(F,E)}}_y \tilde{\tilde{\subseteq}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} \underline{\underline{(F,E)}}$. Now consider $\underline{\underline{(F,E)}}_y = \overline{\overline{(F,E)}}^y \tilde{\tilde{\cap}} \overline{\overline{\tilde{\tilde{Y}} - (F,E)}}^y \Rightarrow$

$[\tilde{\tilde{Y}} \tilde{\tilde{\cap}} \overline{\overline{(F,E)}}] \tilde{\tilde{\cap}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} [\overline{\overline{\tilde{\tilde{Y}} - (F,E)}}]$. Since using the result (i) $[\tilde{\tilde{Y}} \tilde{\tilde{\cap}} \overline{\overline{(F,E)}}] \tilde{\tilde{\cap}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} [\overline{\overline{\tilde{\tilde{Y}} - (F,E)}}]$. (since $\tilde{\tilde{Y}} \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$). $\tilde{\tilde{\subseteq}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} \overline{\overline{(F,E)}}] \tilde{\tilde{\cap}} [\overline{\overline{\tilde{\tilde{X}} - (F,E)}}]$=$\tilde{\tilde{Y}} \tilde{\tilde{\cap}} \underline{\underline{(F,E)}}$. Thus $\underline{\underline{(F,E)}}_y \tilde{\tilde{\subseteq}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} \underline{\underline{(F,E)}}$

This finishes the proof.

3. Binary Soft b-Separation Axioms

In this section binary soft b-separation axioms in Binary Soft Topological Spaces are reflected**.**

Definition 3.1. Let $(U_1, U_2, \tau_\Delta, A)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}_A$ such that $F_e \tilde{\tilde{\neq}} G_e$. Then the binary soft topological space is said to be a binary soft b-τ_o space denoted as b-T_{Δ_o}.If there exists at least one binary soft b-open set (F_1, A) or (F_2, A) such that $F_e \tilde{\tilde{\in}} (F_1, A)$, $G_e \tilde{\tilde{\in}} (F_1, A)$ or $F_e \tilde{\tilde{\in}} (F_2, A)$, $G_e \tilde{\tilde{\in}} (F_2, A)$.

Definition 3.2. Let $(U_1, U_2, \tau_\Delta, A)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}_A$ such that $F_e \tilde{\tilde{\neq}} G_e$. Then the binary soft topological space is said to be a binary soft b-τ_1 space denoted as b-T_{Δ_1}. If there exists at least one binary soft b-open set (F_1, A) or (F_2, A) such that $F_e \tilde{\tilde{\in}} (F_1, A)$, $G_e \tilde{\tilde{\notin}} (F_1, A)$ or $F_e \tilde{\tilde{\in}} (F_2, A)$, $G_e \tilde{\tilde{\notin}} (F_2, A)$.

Definition 3.3. Let $(U_1, U_2, \tau_\Delta, A)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}_A$ such that $F_e \tilde{\tilde{\neq}} G_e$. Then the binary soft topological space is said to be a binary soft b-τ_2 space denoted as b-T_{Δ_2}. If there exists at least one binary soft b-open set (F_1, A) or (F_2, A) such that $F_e \tilde{\tilde{\in}} (F_1, A)$, $H_e \tilde{\tilde{\in}} (F_2, A)$ and$(F_1, E) \tilde{\tilde{\cap}} (F_2, E) = \tilde{\tilde{\varphi}}_A$.

Proposition 3.1. (i) Every b-T_{Δ_1}-space is b-T_{Δ_o}space.
(ii) Every b-T_{Δ_2}-space is b-T_{Δ_1}-space.

Proof. (i) is obvious. (ii) If $(U_1, U_2, \tau_\Delta, A)$ is a T_{Δ_2}-space then by definition for $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}_A$, $F_e \tilde{\tilde{\neq}} G_e$ there exists at least one binary soft b-open set (F_1, A) and (F_2, A) such that $F_e \tilde{\tilde{\in}} (F_1, A)$, $H_e \tilde{\tilde{\in}} (F_2, A)$ and $(F_1, E) \tilde{\tilde{\cap}} (F_2, E) = \tilde{\tilde{\varphi}}_A$. Since $(F_1, E) \tilde{\tilde{\cap}} (F_2, E) = \tilde{\tilde{\varphi}}_A$; $F_e \tilde{\tilde{\notin}} (F_2, A)$ and $G_e \tilde{\tilde{\notin}} (F_1, A)$. Thus it follows that $(U_1, U_2, \tau_\Delta, A)$ is b-T_{Δ_1}space.

Note that every b-T_{Δ_1}space is b-T_{Δ_o}space. Every b-T_{Δ_2}space is b-T_{Δ_1}space.

Proposition 3.2. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $\tilde{\tilde{Y}}$ be a non-empty subset of $\tilde{\tilde{X}}$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}Space then $(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft b-T_{Δ_o}space.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ Now let $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{Y}}$ such that $F_e \tilde{\tilde{\neq}} G_e$. If there exist a binary soft b-open set (F_1, E) in $\tilde{\tilde{X}}$ such that $F_e \tilde{\tilde{\in}} (F_1, E)andG_e \tilde{\tilde{\notin}} (F_1, E)$. Now if $F_e \tilde{\tilde{\in}} \tilde{\tilde{Y}}$implies that$F_e \tilde{\tilde{\in}} \tilde{\tilde{Y}}$. So $F_e \tilde{\tilde{\in}} \tilde{\tilde{Y}}andF_e \tilde{\tilde{\in}} (F_1, E)$. Hence $F_e \tilde{\tilde{\in}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F_1, E) = [^Y(F_1, E)]$, where,$(F_1, E)$ is binary soft b-open set. That is$(F_1, E) \in \tau_\Delta$. Let$G_e \tilde{\tilde{\notin}} (F_1, E)$, this means that $G_e \tilde{\tilde{\notin}} F(\beta)$ for some $\beta \tilde{\tilde{\in}} E$. $G_e \tilde{\tilde{\notin}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F_1, E)_\alpha = Y_\beta(F_1, E)_\alpha$. Therefore, $G_e \tilde{\tilde{\notin}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F_1, E) = [^Y(F_1, E)]$. Similarly, it can prove that if $G_e \tilde{\tilde{\in}} (F_2, E)$ and $F_e \tilde{\tilde{\notin}} (F_2, E)$ then $G_e \tilde{\tilde{\in}} [^Y(F_2, E)]$ and $F_e \tilde{\tilde{\in}} [^Y(F_2, E)]$. Thus $(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft b-T_{Δ_o}space.

Example 3.1. Let $U_1 = \{c_1, c_2, c_3\}, U_2 = \{m_1, m_2\}$ $E = \{e_1, e_2\}$ and

$$\tau_\Delta = \{\tilde{\tilde{X}}, \tilde{\tilde{\varphi}}, \{(e_1(\{c_2\}\{m_2\})),(e_2(\{c_1\}\{m_1\}))\},$$
$$\{(e_1(\{c_1\}\{m_1\})), (e_2(\{c_2\}\{m_2\}))\}, \{(e_1(\{c_1\}\{m_1\}))\}, \{\left(e_1\left(\{\tilde{\tilde{X}}\}\{\tilde{\tilde{X}}\}\right)\right), (e_2(\{c_1\}\{m_1\}))\}\}$$

where

$$(F_1, E) = \{(e_1(\{c_2\}\{m_2\}), (e_2(\{c_1\}\{m_1\}))\},$$
$$(F_2, E) = (e_1(\{c_1\}\{m_1\}), (e_2(\{c_2\}\{m_1\}))\},$$
$$(F_3, E) = \{(e_1(\{c_1\}\{m_1\}))\}$$
$$(F_4, E) = \{\left(e_1\left(\{\tilde{\tilde{X}}\}\{\tilde{\tilde{X}}\}\right)\right), (e_2(\{c_1\}\{m_1\}))\}$$

Clearly $(U_1, U_2, \tau_\Delta, E)$ is binary sot topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$.

Note that

$\tau_{\Delta_1} = \{\tilde{\tilde{X}}, \tilde{\tilde{\varphi}}, \{(e_1(\{c_1\}\{m_1\}))\}, (e_1(\{c_2\}\{m_2\}))\}$
$T_{\Delta_2} = \{\tilde{\tilde{X}}, \tilde{\tilde{\varphi}}, \{(e_2(\{c_1\}\{m_1\}))\}, (e_2(\{c_2\}\{m_2\}))\}$

are binary soft topological spaces on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. There are two pairs of distinct binary soft points namely

$F_{e_1} = \{(e_1(\{c_2\}\{m_2\})\}, G_{e_1} = \{(e_1(\{c_1\}\{m_1\})\}$ and
$F_{e_2} = \{(e_2(\{c_1\}\{m_1\})\}, G_{e_2} = \{(e_2(\{c_2\}\{m_2\})\}$.

Then for binary soft pair $F_{e_1} \neq G_{e_1}$ of points there are binary soft open sets (F_1, E) and (F_2, E) such that$F_{e_1} \tilde{\tilde{\in}} (F_1, E)$, $G_{e_1} \tilde{\tilde{\notin}} (F_1, E)$ and $G_{e_1} \tilde{\tilde{\in}} (F_1, E)$, $F_{e_1} \tilde{\tilde{\notin}} (F_2, E)$. Similarly for the pair $F_{e_2} \neq G_{e_2}$, there are binary soft b-open sets (F_1, E) and (F_2, E) such that $F_{e_2} \tilde{\tilde{\notin}} (F_2, E)$, $G_{e_2} \tilde{\tilde{\in}} (F_2, E)$ and $G_{e_2} \tilde{\tilde{\notin}} (F_1, E)$, $F_{e_2} \tilde{\tilde{\in}} (F_1, E)$. This shows that $(U_1, U_2, \tau_\Delta, E)$ is binary soft space b-T_{Δ_1}-space and hence a binary soft b-T_{Δ_o}space. Note that $(U_1, U_2, \tau_\Delta, E)$ is binary soft b-T_{Δ_2}space.

Proposition 3.3. Let $(U_1, U_2, \tau_\Delta, E)$ is binary soft topological space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Then each binary soft point is binary soft b-closed if and only if $(U_1, U_2, \tau_\Delta, E)$ is binary soft b-T_{Δ_1}space.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ is binary soft topological space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Now to prove let$(U_1, U_2, \tau_\Delta, E)$ is binary soft b-T_{Δ_1}space, suppose binary soft points$F_{e_1} \tilde{\tilde{\equiv}} (F, E)$, $G_{e_1} \tilde{\tilde{\equiv}} (G, E)$ are binary soft b-closed and$F_{e_1} \neq G_{e_1}$. Then $(F, E)^c$ and $(G, E)^c$ are binary soft b-open in$(U_1, U_2, \tau_\Delta, E)$. Then by definition $(F, E)^c = (F^c, E)$ where $F^c(e_1) = \tilde{\tilde{X}} - F(e_1)$ and$(G, E)^c = (G^c, E)$, where$G^c{}_{e_1} = \tilde{\tilde{X}} - G(e_1)$. Since $F(e_1) \tilde{\tilde{\cap}} G(e_1) = \tilde{\tilde{\varphi}}$. This implies $F(e_1) = \tilde{\tilde{X}} - G(e_1) = G^c{}_{e_1} \forall$ e. This implies $F(e_1) = (F, E) \tilde{\tilde{\in}} (G, E)^c$. Similarly $G(e_1) = (G, E) \tilde{\tilde{\in}} (F, E)^c$. Thus we have$(e_1) \tilde{\tilde{\in}} (G, E)^c$, $G(e_1) \tilde{\tilde{\notin}} (G, E)^c$ and $F(e_1) \tilde{\tilde{\notin}} (F, E)^c$, $G\ (e_1) \tilde{\tilde{\in}} (F, E)^c$. This proves that $(U_1, U_2, \tau_\Delta, E)$ is binary soft b-T_1space.

Conversely, let $(U_1, U_2, \tau_\Delta, E)$ is binary soft b-T_{Δ_1}space, to prove that $F(e_1) = (F, E) \tilde{\tilde{\in}} \tilde{\tilde{X}}$ is binary soft pre-closed, we show that $(F, E)^c$ is binary soft b-open in$(U_1, U_2, \tau_\Delta, E)$. Let $G_{e_1} = (G, E) \tilde{\tilde{\in}} (F, E)^c$ is binary soft b-closed. Then$F_{e_1} \neq G_{e_1}$, since $(U_1, U_2, \tau_\Delta, E)$ is binary soft b-T_{Δ_1}space, there exists binary soft b-open set (L, E) such that $G(e_1) \tilde{\tilde{\in}} (L, E) \tilde{\tilde{\subseteq}} (F, E)^c$ and hence $\tilde{\tilde{U}}_{G_{e_1}} \{(L, E),\ G_{e_1} \tilde{\tilde{\in}} (F, E)^c\}$. This proves that $(F, E)^c$ is binary soft b-open in $(U_1, U_2, \tau_\Delta, E)$ that is $F_{e_1} = (F, E)$is binary soft b-closed in$(U_1, U_2, \tau_\Delta, E)$. Which completes the proof.

Proposition 3.4. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $F_e \neq G_e$. If there exist binary soft b-open sets (F_1, E), (F_2, E) such that $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\in}} (F_2, E)^c$, then $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space for each $e \tilde{\tilde{\in}} E$.

Proof. Clearly $G_{e_1} \tilde{\tilde{\in}} (F_1, E)^c = (F_1{}^c, E)$ implies $G_{e_1} \tilde{\tilde{\notin}} (F_2, E)$similarly $F_e \tilde{\tilde{\in}} (F_2, E)^c = (F_2{}^c, E)$ implies $F_e \tilde{\tilde{\notin}} (F_2, E)$. Thus we have $F_e \tilde{\tilde{\in}} (F_1, E)$, $G_e \tilde{\tilde{\notin}} (F_1, E)$ or $G_e \tilde{\tilde{\in}} (F_2, E)$, $F_e \tilde{\tilde{\notin}} (F_2, E)$. This proves $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space. Now for any$\tilde{\tilde{\in}}$ E, $(U_1, U_2, \tau_\Delta, E)$ is a binary soft topological space and $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\in}} (F_1, E)^c$or $G_e \tilde{\tilde{\in}} (F_2, E)$ and $F_e \tilde{\tilde{\notin}} (F_2, E)^c$so that $F_e \tilde{\tilde{\in}} F_1(e)$, $G_e \tilde{\tilde{\notin}} F_1(e)$, $G_e \tilde{\tilde{\in}} F_2(e)$, $G_e \tilde{\tilde{\notin}} F_2(e)$. Thus $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space.

Proposition 3.5. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $F_e \neq G_e$. If there exist binary soft b-open sets (F_1, E), (F_2, E) such that $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\in}} (F_1, E)^c$or $F_e \tilde{\tilde{\in}} (F_2, E)$ and $G_e \tilde{\tilde{\in}} (F_2, E)$, then $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space for each $e \tilde{\tilde{\in}} E$.

Proof. Clearly $G_e \tilde{\tilde{\in}} (F_1, E)^c = (F_1{}^c, E)$ implies $G_e \tilde{\tilde{\notin}} (F_2, E)$ similarly $F_e \tilde{\tilde{\in}} (F_2, E)^c = (F_2{}^c, E)$ implies $F_e \tilde{\tilde{\notin}} (F_2, E)$. Thus we have$F_e \tilde{\tilde{\in}} (F_1, E)$, $G_e \tilde{\tilde{\notin}} (F_1, E)$ or $G_e \tilde{\tilde{\in}} (F_2, E)$, $F_e \tilde{\tilde{\notin}} (F_2, E)$. This proves $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space. Now, for any $\tilde{\tilde{\in}}$ E, $(U_1, U_2, \tau_\Delta, E)$ is a binary soft topological space and $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\in}} (F_1, E)^c$ or $G_e \tilde{\tilde{\in}} (F_2, E)$ and $F_e \tilde{\tilde{\notin}} (F_2, E)^c$. So that $F_e \tilde{\tilde{\in}} F_1(e)$, $G_e \tilde{\tilde{\notin}} F_1(e)$ or $G_e \tilde{\tilde{\in}} F_2(e)$, $F_e \tilde{\tilde{\notin}} F_1(e)$. Thus $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space.

Proposition 3.6. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $F_e \neq G_e$. If there exist binary soft b-open sets (F_1, E), (F_2, E) such that $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\in}} (F_1, E)^c$or $G_e \tilde{\tilde{\in}} (F_2, E)$ and $F_e \tilde{\tilde{\in}} (F_2, E)^c$, then $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_1}space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_1}space for each $e \tilde{\tilde{\in}} E$.

Proof. The proof is similar to the proof **9**.

Now we shill discuss some of the binary soft hereditary properties of b-T_{Δ_i} $(i = 0, 1)$ spaces.

Proposition 3.7. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $\tilde{\tilde{Y}} \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$. Then if $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space then $(U_1, U_2, \tau_{\Delta_y}, E)$ is binary soft b-T_{Δ_o}space.

Proof. $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{Y}}$ such that $F_e \neq G_e$,Then$F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$. Since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_o}space, thus there exists binary soft b-open sets (F, E) and (G, E) in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\tilde{\in}} (F,E)$ and $G_e \tilde{\tilde{\notin}} (F,E)$ or $G_e \tilde{\tilde{\in}} (G,E)$ and $F_e \tilde{\tilde{\notin}} (G,E)$. Therefore $F_e \tilde{\tilde{\in}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F,E) =^{Y} (F,E)$. Similarly I can be shown that if $G_e \tilde{\tilde{\in}} (G,E)$ and $F_e \tilde{\tilde{\notin}} (G,E)$, then $G_e \tilde{\tilde{\in}}^{Y} (G,E)$ and $F_e \tilde{\tilde{\in}}^{Y} (G,E)$ and $F_e \tilde{\tilde{\notin}}^{Y} (G,E)$. Thus $(U_1, U_2, \tau_{\Delta_y}, E)$ is binary soft b-T_{Δ_o}space.

Proposition 3.8. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $\tilde{\tilde{Y}} \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$. Then if $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_1}space then $(U_1, U_2, \tau_{\Delta_y}, E)$ is binary soft b-T_{Δ_1}space.

Proof. The proof is similar to the proof 11.

Proposition 3.9. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft bτ_{Δ_2}space on $\tilde{\tilde{X}}$over $(U_1 \times U_2)$ then $(U_1, U_2, \tau_{\Delta_e}, E)$ is binary soft b-T_{Δ_2}space for each $e \tilde{\tilde{\in}} E$.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. For anye $\tilde{\tilde{\in}} E$, $\tau_{\Delta_e} = \{F(e) : (F,E) \tilde{\tilde{\in}} \tau_\Delta\}$ is a binary soft topology on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Let $x, y \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $x \neq y$, since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_2}space, therefore binary soft points $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $F_e \neq G_e$ and $x \tilde{\tilde{\in}} F(e)$, $y \tilde{\tilde{\in}} G(e)$, there exists binary soft b-open sets (F_1, E) , (F_2, E) such that $F_e \tilde{\tilde{\in}} (F_1, E)$,$G_e \tilde{\tilde{\in}} (F_2, E)$ and $(F_1, E) \tilde{\tilde{\cap}} (F_2, E) = \tilde{\tilde{\varphi}}$. Which implies that $\tilde{\tilde{\in}} F(e) \tilde{\tilde{\subseteq}} F_1(e)$, $y \tilde{\tilde{\in}} G(e) \tilde{\tilde{\subseteq}} F_2(e)$ and $F_1(e) \tilde{\tilde{\cap}} F_2(e) = \tilde{\tilde{\varphi}}$. This proves that $(U_1, U_2, \tau_{\Delta_e}, E)$ is binary soft b-T_{Δ_2}space.

Proposition 3.10. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $\tilde{\tilde{Y}} \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$. Then if $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-τ_{Δ_2}space then $\left(U_1, U_2, \tau_{\Delta_y}, E\right)$ is binary soft b-T_{Δ_2}space and $\left(U_1, U_2, \tau_{\Delta_e}, E\right)$ is binary soft b-T_{Δ_2}space for each $e \tilde{\tilde{\in}} E$.

Proof. Let $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{Y}}$such that $F_e \neq G_e$. Then$F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$. Since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_2}space, thus there exists binary soft b-open sets (F_1, E) and (F_2, E) such that $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\in}} (, E)$ and $(F_1, E) \tilde{\tilde{\cap}} (F_2, E) = \tilde{\tilde{\varphi}}$. Therefore $F_e \tilde{\tilde{\in}} \tilde{\tilde{Y}} \tilde{\tilde{\cap}} (F_2, E) =^{Y} (F_2, E)$ and $^{Y}(F_2, E) \tilde{\tilde{\cap}}^{Y} (F_2, E) = \tilde{\tilde{\varphi}}$. Thus it proves that $(U_1, U_2, \tau_{\Delta_y}, E)$ is binary soft b-T_{Δ_2}space.

Proposition 3.11. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-T_{Δ_2}space and for any two binary soft points $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $F_e \neq G_e$. Then there exist binary soft b-closed sets (F_1, E) and (F_2, E) such that $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\notin}} (F_1, E)$or $G_e \tilde{\tilde{\in}} (F_2, E)$ and $(F_1, E) \tilde{\tilde{\cup}} (F_2, E) = \tilde{\tilde{X}}$.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft b-τ_{Δ_2}space and $F_e, G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ such that $F_e \neq G_e$ there exists binary soft b-open sets (H, E) and (L, E) such that $F_e \tilde{\tilde{\in}} (H, E)$ and $G_e \tilde{\tilde{\in}} (L, E)$ and

$(H, E) \tilde{\tilde{\cap}} (L, E) = \tilde{\tilde{\varphi}}$. Clearly $(H, E) \tilde{\tilde{\subseteq}} (L, E)^c$ and $(L, E) \tilde{\tilde{\subseteq}} (H, E)^c$. Hence $F_e \tilde{\tilde{\in}} (L, E)^c$, put $(L, E)^c = (F_1, E)$ which gives $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \notin (F_1, E)$. Also $G_e \tilde{\tilde{\in}} (F_1, E)^c$, then put $(H, E)^c = (F_2, E)$. Therefore $F_e \tilde{\tilde{\in}} (F_1, E)$ and $G_e \tilde{\tilde{\in}} (F_2, E)$. Moreover, $(F_1, E) \tilde{\tilde{\cup}} (F_2, E) = (L, E)^c \tilde{\tilde{\cup}} (H, E)^c = \tilde{\tilde{X}}$. Which completes the proof.

4. Binary Soft b-T_{Δ_i} (i=4,3) Spaces

In this section binary soft b-separation axioms in Binary Soft Topological Spaces are discussed**.**

In this section, we define binary soft b-regular and binary soft b-T_{Δ_i}- spaces using binary soft points. We also characterize binary soft b-regular and binary soft b-normal spaces. Moreover, we prove that binary soft b-regular and binary soft b-T_{Δ_3} properties are binary soft hereditary, whereas binary soft b- normal and binary soft b-T_{Δ_4} are binary soft b-closed hereditary properties.

Now we define binary soft b-regular space as follows:

Definition 4.1. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Let (F, E) be a binary soft b-closed set in $(U_1, U_2, \tau_\Delta, E)andF_e \tilde{\tilde{\notin}} (F, E)$. If there exists binary soft b-open sets (G, E) and (H, E) such that $F_e \tilde{\tilde{\in}} (G, E)$, $(F, E) \tilde{\tilde{\subseteq}} (H, E)$ and $(F, E) \tilde{\tilde{\cap}} (H, E) = \tilde{\tilde{\varphi}}$, then $(U_1, U_2, \tau_\Delta, E)$ is called a binary soft b- regular space.

Proposition 4.1. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Then the following statements are equivalent:

(i) $(U_1, U_2, \tau_\Delta, E)$ is binary soft b-regular.
(ii) For any binary soft b- open set (F, E) in $(U_1, U_2, \tau_\Delta, E)andG_e \tilde{\tilde{\in}} (F, E)$, there is binary soft b-open set (G, E) containing G_e such that$G_e \tilde{\tilde{\in}} \overline{\overline{(G, E)}} \tilde{\tilde{\subseteq}} (F, E)$.
(iii) Each binary soft point in $(U_1, U_2, \tau_\Delta, E)$ has a binary soft neighborhood base consisting of binary soft b-closed sets.

Proof. (i)⇒ (ii)

Let (F, E) be a binary soft b-open set in $(U_1, U_2, \tau_\Delta, E)andG_e \tilde{\tilde{\in}} (F, E)$. Then $(F, E)^c$ is binary soft b-closed set such that $G_e \tilde{\tilde{\notin}} (F, E)^c$. By he binary soft regularity of $(U_1, U_2, \tau_\Delta, E)$ there are binary soft b-open sets $(F_1, E), (F_2, E)$ such that$G_e \tilde{\tilde{\notin}} (F_1, E)$,$(F, E)^c \tilde{\tilde{\subseteq}} (F_2, E)$ and $(F_1, E) \tilde{\tilde{\cap}} (F_2, E) = \tilde{\tilde{\varphi}}$. Clearly $(F_2, E)^c$ is a binary soft set contained in (F, E). Thus $(F_1, E) \tilde{\tilde{\subseteq}} (F_2, E)^c \tilde{\tilde{\subseteq}} (F, E)$. This gives $\overline{\overline{(F_1, E)}} \tilde{\tilde{\subseteq}} (F_2, E)^c \tilde{\tilde{\subseteq}} (F, E)$, put $(F_1, E) = (G, E)$. Consequently $G_e \tilde{\tilde{\in}} (G, E)$ and $\overline{\overline{(G, E)}} \tilde{\tilde{\subseteq}} (F, E)$. This proves (ii).

(ii)⇒ (iii)

Let $G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$, for binary soft b-open set (F,E) in (U_1,U_2,τ_Δ,E) there is a binary soft b-open set (G,E) containing G_e such that $G_e \tilde{\tilde{\in}} (G,E)$, $\overline{\overline{(G,E)}} \tilde{\tilde{\subseteq}} (F,E)$. Thus for each $G_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$, the sets $\overline{\overline{(G,E)}}$ from a binary soft neighborhood base consisting of binary soft b-closed sets of (U_1,U_2,τ_Δ,E) which proves (iii).

(iii) ⇒ (i)

Let (F,E) be a binary soft b-closed set such that $G_e \tilde{\tilde{\notin}} (F,E)$. Then $(F,E)^c$is a binary soft b-open neighborhood of G_e. By (iii) there is a binary soft b-closed set (F_1,E) which contains G_e and is a binary soft neighborhood of G_e with $(F_1,E) \tilde{\tilde{\subseteq}} (F_1,E)^c$. Then$G_e \tilde{\tilde{\notin}} (F,E)^c$, $(F,E) \tilde{\tilde{\subseteq}} (F_1,E)^c = (F_2,E)$ and $(F_1,E) \tilde{\tilde{\cap}} (F_2,E) = \tilde{\tilde{\varphi}}$. Therefore (U_1,U_2,τ_Δ,E) is binary soft b-regular.

Proposition 4.2. Let (U_1,U_2,τ_Δ,E) be a binary soft b-regular space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Then every binary soft subspace of (U_1,U_2,τ_Δ,E) is binary soft b- regular.

Proof. Let (U_1,U_2,τ_Δ,E) be a binary soft subspace of a binary soft pre-regular space (U_1,U_2,τ_Δ,E). Suppose (F,E) is a binary soft b-closed set in $(U_1,U_2,\tau_{\Delta_y},E)$ and $F_e \tilde{\tilde{\in}} \tilde{\tilde{Y}}$ such that $F_e \tilde{\tilde{\notin}} (F,E)$.Then$(F,E) = (G,E) \tilde{\tilde{\cap}} \tilde{\tilde{Y}}$; Where$(G,E)$ is binary soft b-closed set in (U_1,U_2,τ_Δ,E). Then $F_e \tilde{\tilde{\notin}} (F,E)$, since (U_1,U_2,τ_Δ,E) be a binary soft subspace of a binary soft b-regular, there exists soft disjoint binary b-open sets (F_1,E), (F_2,E) in (U_1,U_2,τ_Δ,E). Then $F_e \tilde{\tilde{\notin}} (G,E)$, Since (U_1,U_2,τ_Δ,E) is binary soft pre- regular, there exist binary soft disjoint binary b-open sets (F_1,E) , (F_2,E) in (U_1,U_2,τ_Δ,E) such that $F_e \tilde{\tilde{\in}} (F_1,E)$, $(G,E) \tilde{\tilde{\in}} (F_2,E)$. Clearly $F_e \tilde{\tilde{\in}} (F_1,E) \tilde{\tilde{\cap}} \tilde{\tilde{Y}} =^Y (F_2,E)$ and $(F,E) \tilde{\tilde{\subseteq}} (F_2,E) \tilde{\tilde{\cap}} \tilde{\tilde{Y}} =^Y (F_2,E)$ such that ${}^Y(F_1,E) \tilde{\tilde{\cap}}^Y(F_2,E) = \tilde{\tilde{\varphi}}$. Therefore it proves that $(U_1,U_2,\tau_{\Delta_y},E)$ is a binary soft b-regular subspace of (U_1,U_2,τ_Δ,E).

Proposition 4.3. Let (U_1,U_2,τ_Δ,E) be a binary soft regular space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. A binary space (U_1,U_2,τ_Δ,E) is binary soft b-regular if and only if for each $F_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$and a binary soft b-closed set (F,E)in (U_1,U_2,τ_Δ,E)such that $F_e \tilde{\tilde{\notin}} (F,E)$ there exist binary soft b-open sets (F_1,E) , (F_2,E)in (U_1,U_2,τ_Δ,E) such that $F_e \tilde{\tilde{\in}} (F_1,E)$, $(F_1,E) \tilde{\tilde{\subseteq}} (F_2,E)$ and $\overline{\overline{(F_1,E)}} \tilde{\tilde{\cap}} \overline{\overline{(F_2,E)}} = \tilde{\tilde{\varphi}}$.

Proof. For each $F_e \tilde{\tilde{\in}} \tilde{\tilde{X}}$ and a binary soft b-closed set (G,E) such that $F_e \tilde{\tilde{\notin}} (F,E)$ by theorem 16there is a binary soft b-open set (G,E) such that $F_e \tilde{\tilde{\in}} (G,E)$, $\overline{\overline{(G,E)}} \tilde{\tilde{\subseteq}} (F_1,E)^c$. Again by theorem16 there is a binary soft b-open (F_1,E) containing F_e such that $\overline{\overline{(F_1,E)}} \tilde{\tilde{\subseteq}} (G,E)$. Let$(F_2,E) = (\overline{\overline{(G,E)}})^c$, then $\overline{\overline{(F_1,E)}} \tilde{\tilde{\subseteq}} (G,E) \tilde{\tilde{\subseteq}} \overline{\overline{(G,E)}} \tilde{\tilde{\subseteq}} (F,E)^c$ Implies $\overline{\overline{(F_1,E)}} \tilde{\tilde{\subseteq}} (\overline{\overline{(G,E)}})^c = (F_2,E)$ or$(F,E) \tilde{\tilde{\subseteq}} (F_2,E)$. Also

$$\overline{\overline{(F_1,E)}} \tilde{\tilde{\cap}} \overline{\overline{(F_2,E)}} = \overline{\overline{(F_1,E)}} \tilde{\tilde{\cap}} \left(\overline{\overline{(G,E)}}\right)^c \tilde{\tilde{\subseteq}} (G,E) \tilde{\tilde{\cap}} \left(\overline{\overline{(G,E)}}\right)^c \tilde{\tilde{\subseteq}} \overline{\overline{(G,E) \tilde{\tilde{\cap}} \left(\overline{\overline{(G,E)}}\right)^c}} = \overline{\overline{\tilde{\tilde{\varphi}}}}$$
$$= \varphi .$$

Thus (F_1, E) , (F_2, E) are the required binary soft b-open sets in$(U_1, U_2, \tau_\Delta, E)$. This proves the necessity. The sufficiency is immediate.

Definition 4.2. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. (F, E) , (G, E) are binary soft b-closed sets over $(U_1 \times U_2)$ such that $(F, E) \tilde{\tilde{\cap}} (G, E) = \tilde{\tilde{\varphi}}$. If there exist binary soft b-open sets (F_1, E), and (F_2, E) such that $(F, E) \tilde{\tilde{\subseteq}} (F_1, E)$, $(G, E) \tilde{\tilde{\subseteq}} (F_2, E)$ and $(F_1, E) \tilde{\tilde{\cap}} (F_2, E) = \varphi$, then $(U_1, U_2, \tau_\Delta, E)$ is called a binary soft b-normal space.

Definition 4.3. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$. Then $(U_1, U_2, \tau_\Delta, E)$ is said to be a binary soft b-τ_{Δ_3}space if it is binary soft b-regular and a binary soft b-τ_{Δ_1} space.

Proposition 4.4. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ and $\tilde{\tilde{Y}} \tilde{\tilde{\subseteq}} \tilde{\tilde{X}}$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary softb-τ_{Δ_3}space then $(U_1, U_2, \tau_{\Delta Y}, E)$ is a binary soft b-τ_{Δ_3}space.

Proof. Straightforward

Definition 4.4. A binary soft topological space $(U_1, U_2, \tau_\Delta, E)$ on $\tilde{\tilde{X}}$ over $(U_1 \times U_2)$ is said to be a binary soft b-τ_{Δ_4}space if it is binary soft b- normal and binary soft b-τ_{Δ_1}space.

Proposition 4.5. A binary soft topological space $(U_1, U_2, \tau_\Delta, E)$ is binary soft b-normal if and only if for soft b-closed set (F, E) and a binary soft b-open set (G, E), such that $(F, E) \tilde{\tilde{\subseteq}} (G, E)$ these exist at least one binary soft b-open set (H, E) containing (F, E) such that $(F, E) \tilde{\tilde{\subseteq}} (H, E) \tilde{\tilde{\subseteq}} \overline{\overline{(H, E)}} \tilde{\tilde{\subseteq}} (G, E)$.

Proof. Let us suppose that $(U_1, U_2, \tau_\Delta, E)$ is a binary soft normal space and (F, E) is any binary soft b- closed subset of $(U_1, U_2, \tau_\Delta, E)$ and $(G, E)$is a binary soft b-open set such that $(F, E) \tilde{\tilde{\subseteq}} (G, E)$. Then $(G, E)^c$ is binary soft b-closed and$(F, E) \tilde{\tilde{\cap}} (G, E)^c = \varphi$. So by supposition, there are binary soft b-open sets (H, E) and (K, E) such that $(F, E) \tilde{\tilde{\subseteq}} (H, E)$, $(G, E)^c \tilde{\tilde{\subseteq}} (K, E)$ and $\tilde{\tilde{\cap}} (K, E) = \tilde{\tilde{\varphi}}$.

Since $(H, E) \tilde{\tilde{\cap}} (K, E) = \tilde{\tilde{\varphi}}$, $(H, E) \tilde{\tilde{\subseteq}}$ $(K, E)^c$. But $(K, E)^c$is binary soft b-closed, so that

$$(F, E) \tilde{\tilde{\subseteq}} (H, E) \tilde{\tilde{\subseteq}} \overline{\overline{(H, E)}} \tilde{\tilde{\subseteq}} (K, E)^c \tilde{\tilde{\subseteq}} (G, E).$$

Hence

$$(F, E) \tilde{\tilde{\subseteq}} (H, E) \tilde{\tilde{\subseteq}} \overline{\overline{(H, E)}} \tilde{\tilde{\subseteq}} (K, E)^c \tilde{\tilde{\subseteq}} (G, E).$$

Conversely, suppose that for every binary soft b-closed set (F, E) and a binary soft b-open set (G, E) such that $(F, E) \tilde{\tilde{\subseteq}} (H, E)$, there is a binary soft b-open set (H, E) such that $(F, E) \tilde{\tilde{\subseteq}} (H, E) \tilde{\tilde{\subseteq}} \overline{\overline{(H, E)}} \tilde{\tilde{\subseteq}} (G, E)$. Let(F_1, E) , (F_2, E) be any two soft disjoint b-closed sets, then $(F_1, E) \tilde{\tilde{\subseteq}} (F_2, E)^c$ where $(F_2, E)^c$binary soft b-open. Hence there is a binary soft b-open set (H, E) such that $(F, E) \tilde{\tilde{\subseteq}} (H, E) \tilde{\tilde{\subseteq}} \overline{\overline{(H, E)}} \tilde{\tilde{\subseteq}} (F_2, E)^c$. But then $(F_2, E) \tilde{\tilde{\subseteq}} (\overline{\overline{(H, E)}})^c$ and $(H, E) \tilde{\tilde{\cap}} (\overline{\overline{(H, E)}})^c \neq \varphi$.

Hence $(F_1, E) \tilde{\tilde{\subseteq}} (H, E)$ and $(F_2, E) \tilde{\tilde{\subseteq}} (\overline{\overline{(H, E)}})^c$ with $(H, E) \tilde{\tilde{\cap}} (\overline{\overline{(H, E)}})^c = \varphi$.

Hence $(U_1, U_2, \tau_\Delta, E)$ is binary soft b- normal space.

5. Conclusion

A soft topology between two sets other than the product soft topology has been touched through proper channel. A soft set with single specific topological structure is unable to shoulder up the responsibility to build the whole theory. So to make the theory strong, some additional structures on soft set has to be introduced. It makes, it more bouncy to grow the soft topological spaces with its infinite applications. In this regards we familiarized soft topological structure known as binary soft b-separation axioms in binary soft topological structure with respect to soft b-open sets.

Topology is the most important branch of pure mathematics which deals with mathematical structures by one way or the others. Recently, many scholars have studied the soft set theory which is coined by Molodtsov [3] and carefully applied to many difficulties which contain uncertainties in our social life. Shabir and Naz familiarized and profoundly studied the foundation of soft topological spaces. They also studied topological structures and displayed their several properties with respect to ordinary points.

In the present work, we constantly study the behavior of binary soft b-separation axioms in binary soft topological spaces with respect to soft points as well as ordinary points. We introduce (b- τ_{Δ_o}, pre- τ_{Δ_1}, b- τ_{Δ_2},b- τ_{Δ_3} and b- τ_{Δ_4}) structures with respect to soft points. In future we will plant these structures in different results. We also planted these axioms to different results. These binary soft b-separation structure would be valuable for the development of the theory of soft in binary soft topology to solve complicated problems, comprising doubts in economics, engineering, medical etc. We also attractively discussed some soft transmissible properties with respect to ordinary as well as soft points. I have fastidiously studied numerous homes on the behalf of Soft Topology. And lastly I determined that soft Topology is totally linked or in other sense we can correctly say that Soft Topology (Separation Axioms) are connected with structure. Provided if it is related with structures then it gives the idea of non-linearity beautifully. In other ways we can rightly say Soft Topology is somewhat directly proportional to non-linearity. Although we use non-linearity in Applied Math. So it is not wrong to say that Soft Topology is applied Math in itself. It means that Soft Topology has the taste of both of pure and applied math. In future I will discuss Separation Axioms in Soft Topology with respect to soft points. We expect that these results in this article will do help the researchers for strengthening the toolbox of soft topological structures. Soft topology provides less information on the behalf of a few choices. The reason for this is that we use a single set in soft topology and in binary soft topology we use double sets .It means that binary soft topology exceeds soft topology in all respect. In the light of above mentioned discussion I can literary say that number of sets is directly proportional to choices. Therefore all mathematicians are kindly informed to emphasize upon it.

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References

[1] A. Acikgöz, N. Tas, Binary Soft Set Theory, Eur. J. of Pure. And Appl. Math., Vol. 9, No. 4, 2016, 452- 463.

[2] S. S. Benchalli, G. Patil, On Binary Soft Separation Axioms in Binary Soft Topological Spaces, Global journal and Applied Mathematics,Vol 13,No.9,2017,5393-5412.

[3] D. Molodtsov, Soft Set Theory First Results. Comput. Math. Appl., 37, 1999, 9 - 31.

[4] D.Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and itsApplication, Nechetkie Sistemy i Myagkie Vychisleniya 1 (1), 2006, 8-39.

[5] M. Shabir, M. Naz, On Some New Operations in Soft Set Theory, Computers and Math. With Appl., 57, 2011, 1786-1799.

[6] S. S. Benchalli, G. Patil, A. S. Dodamani and J. P. Kumar, On Binary Soft Topological Spaces, International Journal of Applied Mathematics 30 (6) (2017), 437-453.

[7] A. Kalaichelvi and P.H. Malini, Application of Fuzzy Soft Sets to Investment Decision Making Problem, International Journal of Mathematical Sciences and Applications 1 (3) (2011) 1583-1586.

[8] N. Y. Özgür and N. Taş, A Note on "Application of Fuzzy Soft Sets to Investment Decision Making Problem", Journal of New Theory 7 (2015) 1-10.

[9] N. Taş, N. Y. Özgür and P. Demir, An Application of Soft Set and Fuzzy Soft Set Theories to Stock Management, Süleyman Demirel University Journal of Natural and Applied Sciences 21 (2) (2017), 791-196.

[10] J. C. R. Alcantud, S. C. Rambaud and M. J. M. Torrecillas, Valuation Fuzzy Soft Sets: A Flexible Fuzzy Soft Set Based Decision Making Procedure for the Valuation of Assets, Symmetry 2017, 9(11), 253; doi:10.3390/sym9110253

[11] N. Çağman, S. Enginoğlu, Soft Matrix Theory and its Decision Making, Computers and Mathematics with Applications 59 (2010) 3308-3314.