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New Types of Some Nano $\mathcal{R}$-Sets

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Abstaract − In this paper, we introduce the notions of nano $\mathcal{R}$-set, nano $\mathcal{R}_r$-set and $\mathcal{R}_r^\star$-set in nano topological spaces and study some of their properties.

Keywords − *Nano $\mathcal{B}$-set, nano t-set, nano $\alpha^\star$-set, nano $\mathcal{R}$-set, nano $\mathcal{R}_r$-set, nano $\mathcal{R}_r^\star$-set.*

1 Introduction

Thivagar and Richard [3] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

Bhuvaneshwari and Gnanapriya [1] introduced and investigated nano g-closed sets in nano topological spaces. Recently, Devi and Bhuvaneswari [6] introduced the notions of nano rg-closed sets. In this paper we introduce the notions of nano $\mathcal{R}$-set, nano $\mathcal{R}_r$-set, nano $\mathcal{R}_r^\star$-set and study some of their properties.

2 Preliminaries

Definition 2.1. [4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by

$L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $n\text{-}int(A)$ and $n\text{-}cl(A)$, respectively.

Definition 2.3. A subset H of a space $(U, \mathcal{N})$ is called nano regular-pen set [3] if $H = n\text{-}int(n\text{-}cl(H))$.

The complement of the above mentioned set is called their respective closed set.

Definition 2.4. [2] A subset H of a space $(U, \mathcal{N})$ is called:

1. nano t-set (briefly, nt-set) if $n\text{-}int(H) = n\text{-}int(n\text{-}cl(H))$.

2. nano $\mathcal{B}$-set (briefly, $n\mathcal{B}$-set) if $H = P \cap Q$, where P is n-open and Q is nt-set.

Definition 2.5. [5] A subset H of a space $(U, \mathcal{N})$ is called a nano $\alpha^\star$-set (briefly, $n\alpha^\star$-set) if $n\text{-}int(n\text{-}cl(n\text{-}int(H))) = n\text{-}int(H)$.

Definition 2.6. A subset H of a space $(U, \mathcal{N})$ is called;

1. nano g-closed (briefly, ng-closed) [1] if $n\text{-}cl(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.

2. nano rg-closed set (briefly, nrg-closed) [6] if $n\text{-}cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano regular-open.

3 Properties of Some Nano $\mathcal{R}$-Sets

Definition 3.1. A subset H of a space $(U, \mathcal{N})$ is called;

1. nano $\mathcal{R}$-set (briefly, $n\mathcal{R}$-set) if $H = P \cap K$ where P is ng-open and K is nt-set.
2. nano $\mathcal{R}_r$-set (briefly, $n\mathcal{R}_r$-set) if $H = P \cap K$ where P is nrg-open and K is nt-set.
3. nano $\mathcal{R}_r^\star$-set (briefly, $n\mathcal{R}_r^\star$-set) if $H = P \cap K$ where P is nrg-open and K is $n\alpha^\star$-set.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$. Then the nano topology $\mathcal{N} = \{\phi, \{d\}, \{b, c\}, \{b, c, d\}, U\}$.

1. then $\{a\}$ is $n\mathcal{R}$-set.
2. then $\{a, b\}$ is $n\mathcal{R}_r$-set.
3. then $\{a, b, c\}$ is $n\mathcal{R}_r^\star$-set.

Theorem 3.3. In a space $(U, \mathcal{N})$, for a subset H, the following relations hold.

1. H is $n\mathcal{B}$-set $\Rightarrow$ H is $n\mathcal{R}$-set.
2. H is nt-set $\Rightarrow$ H is $n\mathcal{R}_r$-set.
3. H is nrg-open set $\Rightarrow$ H is $n\mathcal{R}_r$-set.
4. H is $n\alpha^\star$-set $\Rightarrow$ H is $n\mathcal{R}_r^\star$-set
5. H is nrg-open set $\Rightarrow$ H is $n\mathcal{R}_r^\star$-set.
6. H is nt-set $\Rightarrow$ H is $n\mathcal{R}_r^\star$-set.
7. H is $n\mathcal{R}_r$-set $\Rightarrow$ H is $n\mathcal{R}_r^\star$-set.

Proof.

1. Since every n-open set is ng-open, every $n\mathcal{B}$-set is a $n\mathcal{R}$-set.
2. Let H be a nt-set in U. Then $H = U \cap H$ where U is clearly nrg-open in U. Therefore, H is $n\mathcal{R}_r$-set in U.
3. Let H be a nrg-open set in U. Then $H = H \cap U$ where U is clearly a nt-set in U. Therefore, H is $n\mathcal{R}_r$-set in U.
4. Let H be a $n\alpha^\star$-set in U. Then $H = U \cap H$ where U is clearly nrg-open in U. Therefore, H is a $n\mathcal{R}_r^\star$-set in U.
5. Let H be a nrg-open set in U. Then $H = H \cap U$ where U is clearly a $n\alpha^\star$-set in U. Therefore, H is a $n\mathcal{R}_r^\star$-set in U.
6. Let H be a nt-set in U. Since every nt-set is $n\alpha^\star$-set in U. So, H is $n\alpha^\star$-set in U. By (4), U is a $n\mathcal{R}_r^\star$-set in U.

7. Let H be a $n\mathcal{R}_r$-set in U. Then $H = P \cap Q$ where P is nrg-open in U and Q is a nt-set in U. Since every nt-set in U is a $n\alpha^\star$-set in U, Q is a $n\alpha^\star$-set in U. Therefore, H is a $n\mathcal{R}_r^\star$-set in U.

Remark 3.4. These relations are shown in the diagram.

$$\begin{array}{ccccccc}
 & & & & n\mathcal{R}\textbf{-set} & \longleftarrow & n\mathcal{B}\textbf{-set} \\
 & & & & \downarrow & & \\
 & & nt\textbf{-set} & \longrightarrow & n\mathcal{R}_r\textbf{-set} & & \\
 & & \downarrow & \swarrow & \uparrow & & \\
n\alpha^\star\textbf{-set} & \longrightarrow & n\mathcal{R}_r^\star\textbf{-set} & \longleftarrow & nrg\textbf{-open} & &
\end{array}$$

The converses of each statement in Theorem 3.3 are not true as shown in the following Example.

Example 3.5. Let $U = \{p, q, r\}$ with $U/R = \{\{p, q\}, \{r\}\}$ and $X = \{p\}$. Then the nano topology $\mathcal{N} = \{\phi, \{p, q\}, U\}$. Then $H = \{p\}$ is $n\mathcal{R}$-set but not $n\mathcal{B}$-set.

Example 3.6. In Example 3.2,

1. $\{b\}$ is $n\mathcal{R}_r$-set but not nt-set.

2. $\{a, d\}$ is $n\mathcal{R}_r$-set but not nrg-open.

3. $\{b, c, d\}$ is $n\mathcal{R}_r^\star$-set but not $n\alpha^\star$-set.

4. $\{a, d\}$ is $n\alpha^\star$-set, so $n\mathcal{R}_r^\star$-set. But {a,d} is not nrg-open.

5. $\{a, b\}$ is $n\mathcal{R}_r^\star$-set but not nt-set.

6. $\{a, b, d\}$ is $n\mathcal{R}_r^\star$-set but not $n\mathcal{R}_r$-set.

Remark 3.7. In a space $(U, \mathcal{N})$,

1. the intersection of two $n\mathcal{R}$-sets are $n\mathcal{R}$-set.

2. the intersection of two $n\mathcal{R}_r$-sets are $n\mathcal{R}_r$-set.

3. the intersection of two $n\mathcal{R}_r^\star$-sets are $n\mathcal{R}_r^\star$-set.

4. the union of two $n\mathcal{R}$-sets but not $n\mathcal{R}$-set.

5. the union of two $n\mathcal{R}_r$-sets but not $n\mathcal{R}_r$-set.

Example 3.8. In Example 3.2,

1. then $H = \{a, d\}$ and $Q = \{b, d\}$ is $n\mathcal{R}$-sets, $n\mathcal{R}_r$-sets and $n\mathcal{R}_r^\star$-sets. But $H \cap Q = \{d\}$ is $n\mathcal{R}$-set, $n\mathcal{R}_r$-set and $n\mathcal{R}_r^\star$-set.

2. then $H = \{a\}$ and $Q = \{b\}$ is $n\mathcal{R}$-sets. But $H \cup Q = \{a, b\}$ is not $n\mathcal{R}$-set.

3. then $H = \{a, b\}$ and $Q = \{d\}$ is $n\mathcal{R}_r$-sets. But $H \cup Q = \{a, b, d\}$ is not $n\mathcal{R}_r$-sets.

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