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## On Grill $S_p$ -Open Set in Grill Topological Spaces

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**Abstract** - In this paper, we introduce a new type of grill set namely;  $G_{S_p}$ -open sets, which is analogous to the  $G$ -semiopen sets in a grill topological space  $(X, \tau, G)$ . Further, we define  $G_{S_p}$ -continuous and  $G_{S_p}$ -open functions by using a  $G_{S_p}$ -open set and we investigate some of their important properties.

**Keywords** -  $G_{S_p}$ -open set,  $G_{S_p}O(X)$ ,  $G_{S_p}$ -continuous function,  $G_{S_p}$ -open function.

### 1. Introduction and Preliminaries

Choquet [2] introduced the concept of grill on a topological space and the idea of grills has shown to be an essential tool for studying some topological concepts. A collection  $G$  of nonempty subsets of a topological space  $(X, \tau)$  is called a grill on  $X$  if (i)  $A \in G$  and  $A \subseteq B$  implies that  $B \in G$ , and (ii)  $A, B \subseteq X$  and  $A \cup B \in G$  implies that  $A \in G$  or  $B \in G$ . A triple  $(X, \tau, G)$  is called a grill topological space.

Roy and Mukherjee [17] defined a unique topology by a grill and they studied topological concepts. For any point  $x$  of a topological space  $(X, \tau)$ ,  $\tau(x)$  denotes the collection of all open neighborhoods of  $x$ . A mapping  $\varphi : P(X) \rightarrow P(X)$  is defined as  $\varphi(A) = \{x \in X : A \cap U \in G \text{ for all } U \in \tau(x)\}$  for each  $A \in P(X)$ . A mapping  $\psi : P(X) \rightarrow P(X)$  is defined as  $\psi(A) = A \cup \varphi(A)$  for all  $A \in P(X)$ . The map  $\psi$  satisfies Kuratowski closure axioms:

- (i)  $\psi(\emptyset) = \emptyset$ ,
- (ii) if  $A \subseteq B$ , then  $\psi(A) \subseteq \psi(B)$ ,
- (iii) if  $A \subseteq X$ , then  $\psi(\psi(A)) = \psi(A)$ , and

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- (iv) if  $A, B \subseteq X$ , then  $\psi(A \cup B) = \psi(A) \cup \psi(B)$ .

Corresponding to a grill  $G$  on a topological space  $(X, \tau)$ , there exists a unique topology  $\tau_G$  (say) on  $X$  given by  $\tau_G = \{U \subseteq X : \psi(X - U) = X - U\}$ , where for any  $A \subseteq X$ ,  $\psi(A) = A \cup \varphi(A) = \tau_G\text{-cl}(A)$  and  $\tau \subseteq \tau_G$ .

The concept of decompositions of continuity on a grill topological space and some classes of sets were defined with respect to grill (see [3, 7, 10] for details). A subset  $A$  in  $X$  is said to be

- (i)  $\varphi$ -open if  $A \subseteq \text{int}(\varphi(A))$ ,
- (ii)  $G$ - $\alpha$ .open if  $A \subseteq \text{int}(\psi(\text{int}(A)))$ ,
- (iii)  $G$ -preopen if  $A \subseteq \text{int}(\psi(A))$ ,
- (iv)  $G$ -semiopen if  $A \subseteq \psi(\text{int}(A))$ ,
- (v)  $G$ - $\beta$ .open if  $A \subseteq \text{cl}(\text{int}(\psi(A)))$ .

The family of all  $G$ - $\alpha$ .open (resp.  $G$ -preopen,  $G$ -semiopen,  $G$ - $\beta$ .open) sets in a grill topological space  $(X, \tau, G)$  is denoted by  $G\alpha O(X)$  (rep.  $GPO(X)$ ,  $GSO(X)$ ,  $G\beta O(X)$ ). A function  $f: (X, \tau, G) \rightarrow (Y, \sigma)$  is said to be  $G$ -semicontinuous if  $f^{-1}(V) \in GSO(X)$  for each  $V \in \sigma$ .

Mashhour et al. [14] introduced a class of preopen sets and he defined pre interior and pre closure in a topological space. A subset  $A$  in  $X$  is said to be preopen if  $A \subseteq \text{int}(\text{cl}(A))$  and  $PO(X)$  denotes the family of preopen sets. For any subset  $A$  of  $X$ , (i)  $\text{pint}(A) = \cup\{U : U \in PO(X) \text{ and } U \subseteq A\}$ ; (ii)  $\text{pcl}(A) = \cap\{F : X - F \in PO(X) \text{ and } A \subseteq F\}$ .

In this paper, we define a  $Gs_p$ -open set in a grill topological space  $(X, \tau, G)$  and we study some of its basic properties. Moreover, we define  $Gs_p$ -continuous,  $Gs_p$ -open,  $Gs_p$ -closed and  $Gs_p^*$ -continuous functions on a grill topological space  $(X, \tau, G)$  and we discuss some of their essential properties.

**Proposition 1.1.** [17] Let  $(X, \tau, G)$  be a grill topological space. Then for all  $A, B \subseteq X$ :

- (i)  $A \subseteq B$  implies that  $\varphi(A) \subseteq \varphi(B)$ ;
- (ii)  $\varphi(A \cup B) = \varphi(A) \cup \varphi(B)$ ;
- (iii)  $\varphi(\varphi(A)) \subseteq \varphi(A) = \text{cl}(\varphi(A)) \subseteq \text{cl}(A)$ .

## 2. $Gs_p$ -Open Sets

**Definition 2.1.** Let  $(X, \tau, G)$  be a grill topological space and let  $A$  be a subset  $A$  of  $X$ . Then  $A$  is said to be  $Gs_p$ -open if and only if there exist a  $U \in PO(X)$  such that  $U \subseteq A \subseteq \psi(U)$ . A set  $A$  of  $X$  is  $Gs_p$ -closed if its complement  $X - A$  is  $Gs_p$ -open. The family of all  $Gs_p$ -open (resp.  $Gs_p$ -closed) sets is denoted by  $Gs_p O(X)$  (resp.  $Gs_p C(X)$ ).

**Example 2.1.** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$  and  $G = \{\{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $Gs_p O(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ .

**Theorem 2.1.** Let  $(X, \tau, G)$  be a grill topological space and let  $A \subseteq X$ . Then  $A \in Gs_pO(X)$  if and only if  $A \subseteq \psi(\text{pint}(A))$ .

Proof. If  $A \in Gs_pO(X)$ , then there exist a  $U \in PO(X)$  such that  $U \subseteq A \subseteq \psi(U)$ . But  $U \subseteq A$  implies that  $U \subseteq \text{pint}(A)$ . Hence  $\psi(U) \subseteq \psi(\text{pint}(A))$ . Therefore  $A \subseteq \psi(\text{pint}(A))$ . Conversely, let  $A \subseteq \psi(\text{pint}(A))$ . To prove that  $A \in Gs_pO(X)$ , take  $U = \text{pint}(A)$ , then  $U \subseteq A \subseteq \psi(U)$ . Hence  $A \in Gs_pO(X)$ .

**Corollary 2.1.** If  $A \subseteq X$ , then  $A \in Gs_pO(X)$  if and only if  $\psi(A) = \psi(\text{pint}(A))$ .

Proof. Let  $A \in Gs_pO(X)$ . Then as  $\psi$  is monotonic and idempotent,  $\psi(A) \subseteq \psi(\psi(\text{pint}(A))) = \psi(\text{pint}(A)) \subseteq \psi(A)$  implies that  $\psi(A) = \psi(\text{pint}(A))$ . The converse is obvious.

**Theorem 2.2.** Let  $(X, \tau, G)$  be a grill topological space. If  $A \in Gs_pO(X)$  and  $B \subseteq X$  such that  $A \subseteq B \subseteq \psi(\text{pint}(A))$ , then  $B \in Gs_pO(X)$ .

Proof. Given  $A \in Gs_pO(X)$ . Then by Theorem 2.1,  $A \subseteq \psi(\text{pint}(A))$ . But  $A \subseteq B$  implies that  $\text{pint}(A) \subseteq \text{pint}(B)$  and hence by Theorem 2.4[17],  $\psi(\text{pint}(A)) \subseteq \psi(\text{pint}(B))$ . Therefore  $B \subseteq \psi(\text{pint}(A)) \subseteq \psi(\text{pint}(B))$ . Hence  $B \in Gs_pO(X)$ .

**Corollary 2.2.** If  $A \in Gs_pO(X)$  and  $B \subseteq X$  such that  $A \subseteq B \subseteq \psi(A)$ , then  $B \in Gs_\alpha O(X)$ .

Proof. Follows from the Theorem 2.2 and Corollary 2.1.

**Proposition 2.1.** If  $U \in PO(X)$ , then  $U \in Gs_pO(X)$ .

Proof. Let  $U \in PO(X)$ , it implies that  $U = \text{pint}(U) \subseteq \psi(\text{pint}(U))$ . Hence  $U \in Gs_pO(X)$ .

Note that the converse of the above proposition need not be true. Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$  and  $G = \{\{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $PO(X) = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$  and  $Gs_pO(X) = \{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$ . Here  $\{b, d\}$  and  $\{a, b, d\}$  are  $Gs_p$ -open sets but not preopen.

**Theorem 2.3.** Let  $(X, \tau, G)$  be a grill topological space. If  $A \in GSO(X)$ , then  $A \in Gs_pO(X)$ .

Proof. Given  $A \in GSO(X)$ . Then  $A \subseteq \psi(\text{int}(A))$ . Since  $\text{int}(A) \subseteq \text{pint}(A)$ , we have that  $\psi(\text{int}(A)) \subseteq \psi(\text{pint}(A))$  (by Theorem 2.4[17]). Hence  $A \subseteq \psi(\text{pint}(A))$  and thus  $A \in Gs_pO(X)$ .

Note that the converse of the above theorem need not be true. By Example 2.1, we have that  $GSO(X) = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$ . Therefore  $\{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}$  and  $\{b, c, d\}$  are  $Gs_p$ -open sets but not G-semiopen.

**Proposition 2.2.** If  $PO(X) = \tau$ , then  $G_{S_p}O(X) = GSO(X)$ .

Proof. By Theorem 2.3,  $GSO(X) \subseteq G_{S_p}O(X)$ . Let  $A \in G_{S_p}O(X)$ . Then by Theorem 2.1,  $A \subseteq \psi(\text{pint}(A))$ . Since  $PO(X) = \tau$ , we have that  $\text{pint}(A) = \text{int}(A)$  implies that  $A \subseteq \psi(\text{pint}(A)) = \psi(\text{int}(A))$  and hence  $A \in GSO(X)$ . Thus  $G_{S_p}O(X) \subseteq GSO(X)$ .

**Theorem 2.4.** Let  $(X, \tau, G)$  be a grill topological space.

(i) If  $A_i \in G_{S_p}O(X)$  for each  $i \in J$ , then  $\cup_{i \in J} A_i \in G_{S_p}O(X)$ ;

(ii) If  $A \in G_{S_p}O(X)$  and  $U \in PO(X)$ , then  $A \cap U \in G_{S_p}O(X)$ .

Proof. (i) Since  $A_i \in G_{S_p}O(X)$ , we have that  $A_i \subseteq \psi(\text{pint}(A_i))$  for each  $i \in J$ . Thus, we obtain  $A_i \subseteq \psi(\text{pint}(A_i)) \subseteq \psi(\text{pint}(\cup_{i \in J} A_i))$  and hence  $\cup_{i \in J} A_i \subseteq \psi(\text{pint}(\cup_{i \in J} A_i))$ . This shows that  $\cup_{i \in J} A_i \in G_{S_p}O(X)$ .

(ii) Let  $A \in G_{S_p}O(X)$  and  $U \in PO(X)$ . Then  $A \subseteq \psi(\text{pint}(A))$  and  $\text{pint}(U) = U$ . Now,  $A \cap U \subseteq \psi(\text{pint}(A)) \cap U = (\text{pint}(A) \cup \varphi(\text{pint}(A))) \cap U = (\text{pint}(A) \cap U) \cup (\varphi(\text{pint}(A)) \cap U) \subseteq \text{pint}(A \cap U) \cup \varphi(\text{pint}(A) \cap U)$  (by Theorem 2.10[17])  $= \text{pint}(A \cap U) \cup \varphi(\text{pint}(A \cap U)) = \psi(\text{pint}(A \cap U))$ . Therefore  $A \cap U \in G_{S_p}O(X)$ .

**Remark 2.1.** The following example shows that if  $A, B \in G_{S_p}O(X)$ , then  $A \cap B \notin G_{S_p}O(X)$ .

From Example 2.1, take  $A = \{b, c\}$  and  $B = \{c, d\}$ , then  $A, B \in G_{S_p}O(X)$  but  $A \cap B = \{c\} \notin G_{S_p}O(X)$ .

**Theorem 2.5.** Let  $(X, \tau, G)$  be a grill topological space and  $A \subseteq X$ . If  $A \in G_{S_p}C(X)$ , then  $\text{pint}(\psi(A)) \subseteq A$ .

Proof. Suppose  $A \in G_{S_p}C(X)$ . Then  $X - A \in G_{S_p}O(X)$  and hence  $X - A \subseteq \psi(\text{pint}(X - A)) \subseteq \text{pcl}(\text{pint}(X - A)) = X - \text{pint}(\text{pcl}(A)) \subseteq X - \text{pint}(\psi(A))$ , implies that  $\text{pint}(\psi(A)) \subseteq A$ .

**Theorem 2.6.** Let  $(X, \tau, G)$  be a grill topological space and  $A \subseteq X$  such that  $X - \text{pint}(\psi(A)) = \psi(\text{pint}(X - A))$ . Then  $A \in G_{S_p}C(X)$  if and only if  $\text{pint}(\psi(A)) \subseteq A$ .

Proof. Necessary part is proved by Theorem 2.5. Conversely, suppose that  $\text{pint}(\psi(A)) \subseteq A$ . Then  $X - A \subseteq X - \text{pint}(\psi(A)) = \psi(\text{pint}(X - A))$ , implies that  $X - A \in G_{S_p}O(X)$ . Hence  $A \in G_{S_p}C(X)$ .

**Definition 2.2.** Let  $(X, \tau, G)$  be a grill topological space and  $A \subseteq X$ . Then

(i)  $G_{S_p}$ -interior of  $A$  is defined as union of all  $G_{S_p}$ -open sets contained in  $A$ .

Thus  $G_{S_p}\text{int}(A) = \cup\{U : U \in G_{S_p}O(X) \text{ and } U \subseteq A\}$ ;

(ii)  $G_{S_p}$ -closure of  $A$  is defined as intersection of all  $G_{S_p}$ -closed sets containing  $A$ .

Thus  $G_{S_p}\text{cl}(A) = \cap\{F : X - F \in G_{S_p}O(X) \text{ and } A \subseteq F\}$ .

**Theorem 2.7.** Let  $(X, \tau, G)$  be a grill topological space and  $A \subseteq X$ . Then

- (i)  $Gs_p \text{int}(A)$  is a  $Gs_p$ -open set contained in  $A$ ;
- (ii)  $Gs_p \text{cl}(A)$  is a  $Gs_p$ -closed set containing  $A$ ;
- (iii)  $A$  is  $Gs_p$ -closed if and only if  $Gs_p \text{cl}(A) = A$ ;
- (iv)  $A$  is  $Gs_p$ -open if and only if  $Gs_p \text{int}(A) = A$ ;
- (v)  $Gs_p \text{int}(A) = X - Gs_p \text{cl}(X - A)$ ;
- (vi)  $Gs_p \text{cl}(A) = X - Gs_p \text{int}(X - A)$ .

Proof. Follows from the Definition 2.15 and Theorem 2.4(i).

**Theorem 2.8.** Let  $(X, \tau, G)$  be a grill topological space and  $A, B \subseteq X$ . Then the following are hold: (i) If  $A \subseteq B$ , then  $Gs_p \text{int}(A) \subseteq Gs_p \text{int}(B)$ ;

- (ii)  $Gs_p \text{int}(A \cup B) \supseteq Gs_p \text{int}(A) \cup Gs_p \text{int}(B)$ ;
- (iii)  $Gs_p \text{int}(A \cap B) = Gs_p \text{int}(A) \cap Gs_p \text{int}(B)$ .

Proof. Follows from the Theorem 2.8.

**Definition 2.3.** A function  $f: (X, \tau, G) \rightarrow (Y, \sigma)$  is said to be  $Gs_p$ -continuous if  $f^{-1}(V) \in Gs_p O(X)$  for each  $V \in PO(Y)$ .

**Example 2.2.** Let  $X = \{a, b, c, d\}$ ,  $Y = \{1, 2, 3, 4\}$ ,  $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}\}$ ,  $\sigma = \{\emptyset, Y, \{1, 2\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}\}$  and  $G = \{\{a, b, c\}, X\}$ . Then  $Gs_p O(X) = P(X)$  and  $PO(Y) = \{\emptyset, Y, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$ . Define  $f: (X, \tau, G) \rightarrow (Y, \sigma)$  by  $f(a) = 2$ ,  $f(b) = 1$ ,  $f(c) = 4$  and  $f(d) = 3$ . Then inverse image of every preopen sets in  $Y$  is  $Gs_p$ -open in  $X$ . Hence  $f$  is  $Gs_p$ -continuous.

**Remark 2.2.** The concepts of G-semicontinuous and  $Gs_p$ -continuous are independent.

(i) From Example 2.2, we have that  $GSO(X) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$  and the function  $f$  is  $Gs_p$ -continuous. Also  $f^{-1}(\{1, 2, 3\}) = \{a, b, d\}$  is not G-semiopen in  $X$  for the open set  $\{1, 2, 3\}$  of  $Y$ . Hence  $f$  is not G-semicontinuous.

(ii) Let  $X = \{a, b, c, d\}$ ,  $Y = \{1, 2, 3, 4\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ ,  $\sigma = \{\emptyset, Y, \{1, 2\}, \{3, 4\}\}$  and  $G = \{\{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ . Then  $GSO(X) = \tau$ ,  $Gs_p O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$  and  $PO(Y) = P(Y)$ . Define  $f: (X, \tau, G) \rightarrow (Y, \sigma)$  by  $f(a) = 4$ ,  $f(b) = 3$ ,  $f(c) = 2$  and  $f(d) = 1$ . Then the function  $f$  is G-semicontinuous. Also the inverse image  $f^{-1}(\{3\}) = \{b\}$  is not  $Gs_p$ -open in  $X$  for the preopen set  $\{3\}$  of  $Y$ . Hence  $f$  is not  $Gs_p$ -continuous.

From (i) and (ii), we got the concepts of G-semicontinuous and  $Gs_p$ -continuous are independent.

**Theorem 2.9.** For a function  $f: (X, \tau, G) \rightarrow (Y, \sigma)$ , the following are equivalent:

- (i)  $f$  is  $Gs_p$ -continuous;
- (ii) For each  $F \in PC(Y)$ ,  $f^{-1}(F) \in Gs_pC(X)$ ;
- (iii) For each  $x \in X$  and each  $V \in PO(Y)$  containing  $f(x)$ , there exists a  $U \in Gs_pO(X)$  containing  $x$  such that  $f(U) \subseteq V$ .

Proof. (i)  $\Leftrightarrow$  (ii): It is obvious.

(i)  $\Rightarrow$  (iii): Let  $V \in PO(Y)$  and  $f(x) \in V (x \in X)$ . Then by (i),  $f^{-1}(V) \in Gs_pO(X)$  containing  $x$ . Taking  $f^{-1}(V) = U$ , we have that  $x \in U$  and  $f(U) \subseteq V$ .

(iii)  $\Rightarrow$  (i): Let  $V \in PO(Y)$  and  $x \in f^{-1}(V)$ . Then  $f(x) \in V \in PO(Y)$  and hence by (iii), there exists a  $U \in Gs_pO(X)$  containing  $x$  such that  $f(U) \subseteq V$ . Thus, we obtain  $x \in U \subseteq \psi(\text{pint}(U)) \subseteq \psi(\text{pint}(f^{-1}(V)))$ . This shows that  $f^{-1}(V) \subseteq \psi(\text{pint}(f^{-1}(V)))$ . Hence  $f$  is  $Gs_p$ -continuous.

**Theorem 2.10.** A function  $f: (X, \tau, G) \rightarrow (Y, \sigma)$  is  $Gs_p$ -continuous if and only if the graph function  $g : X \rightarrow X \times Y$ , defined by  $g(x) = (x, f(x))$  for each  $x \in X$ , is  $Gs_p$ -continuous.

Proof. Suppose that  $f$  is  $Gs_p$ -continuous. Let  $x \in X$  and  $W \in PO(X \times Y)$  containing  $g(x)$ . Then there exist a  $U \in PO(X)$  and  $V \in PO(Y)$  such that  $g(x) = (x, f(x)) \in U \times V \subseteq W$ . Since  $f$  is  $Gs_p$ -continuous, there exists a  $G \in Gs_pO(X)$  containing  $x$  such that  $f(G) \subseteq V$ . By Theorem 2.4(b),  $G \cap U \in Gs_pO(X)$  and  $g(G \cap U) \subseteq U \times V \subseteq W$ . This shows that  $g$  is  $Gs_p$ -continuous. Conversely, suppose that  $g$  is  $Gs_p$ -continuous. Let  $x \in X$  and  $V \in \alpha(Y)$  containing  $f(x)$ . Then  $X \times V \in PO(X \times Y)$  and by  $Gs_p$ -continuity of  $g$ , there exists a  $U \in Gs_pO(X)$  containing  $x$  such that  $g(U) \subseteq X \times V$ . Thus we have that  $f(U) \subseteq V$  and hence  $f$  is  $G-s_p$ -continuous.

**Definition 2.3.** Let  $(X, \tau)$  be a topological space and  $(Y, \sigma, G)$  a grill topological space. A function  $f: (X, \tau) \rightarrow (Y, \sigma, G)$  is said to be  $Gs_p$ -open (resp.  $Gs_p$ -closed) if for each  $U \in PO(X)$  (resp. for each  $U \in PC(X)$ ),  $f(U)$  is  $Gs_p$ -open (resp.  $Gs_p$ -closed) in  $(Y, \sigma, G)$ .

**Theorem 2.11.** A function  $f: (X, \tau) \rightarrow (Y, \sigma, G)$  is  $Gs_p$ -open if and only if for each  $x \in X$  and each pre-neighbourhood  $U$  of  $x$ , there exists a  $V \in Gs_pO(Y)$  such that  $f(x) \in V \subseteq f(U)$ .

Proof. Suppose that  $f$  is a  $G-s_p$ .open function and let  $x \in X$ . Also let  $U$  be any pre-neighbourhood of  $x$ . Then there exists  $G \in PO(X)$  such that  $x \in G \subseteq U$ . Since  $f$  is  $Gs_p$ -open,  $f(G) = V$  (say)  $\in Gs_pO(Y)$  and  $f(x) \in V \subseteq f(U)$ . Conversely, suppose that  $U \in PO(X)$ . Then for each  $x \in U$ , there exists a  $V_x \in Gs_pO(Y)$  such that  $f(x) \in V_x \subseteq f(U)$ . Thus  $f(U) = \cup\{V_x : x \in U\}$  and hence by Theorem 2.4(a),  $f(U) \in Gs_pO(Y)$ . This shows that  $f$  is  $Gs_p$ -open.

**Theorem 2.12.** Let  $f: (X, \tau) \rightarrow (Y, \sigma, G)$  be a  $G-s_p$ .open function. If  $V \subseteq Y$  and  $F \in PC(X)$  containing  $f^{-1}(V)$ , then there exists a  $H \in Gs_pO(Y)$  containing  $V$  such that  $f^{-1}(H) \subseteq F$ .

Proof. Suppose that  $f$  is  $G\text{-}S_p$ -open. Let  $V \subseteq Y$  and  $F \in PC(X)$  containing  $f^{-1}(V)$ . Then  $X - F \in PO(X)$  and by  $G\text{-}S_p$ -openness of  $f$ ,  $f(X - F) \in G\text{-}S_pO(X)$ . Thus  $H = Y - f(X - F) \in G\text{-}S_pC(Y)$  consequently  $f^{-1}(V) \subseteq F$  implies that  $V \subseteq H$ . Further, we obtain that  $f^{-1}(H) \subseteq F$ .

**Theorem 2.13.** For any bijection  $f: (X, \tau) \rightarrow (Y, \sigma, G)$ , the following are equivalent:

- (i)  $f^{-1}: (Y, \sigma, G) \rightarrow (X, \tau)$  is  $G\text{-}S_p$ -continuous;
- (ii)  $f$  is  $G\text{-}S_p$ -open;
- (iii)  $f$  is  $G\text{-}S_p$ -closed.

Proof. It is obvious.

**Definition 2.4.** Let  $(X, \tau, G)$  be a grill topological space. A subset  $A$  of  $X$  is said to be a  $G\text{-}S_p^*$ -set if  $A = U \cap V$ , where  $U \in PO(X)$  and  $\psi(\text{pint}(V)) = \text{pint}(V)$ .

**Theorem 2.14.** Let  $(X, \tau, G)$  be a grill topological space and let  $A \subseteq X$ . Then  $A \in PO(X)$  if and only if  $A \in G\text{-}S_pO(X)$  and  $A$  is  $G\text{-}S_p^*$ -set in  $(X, \tau, G)$ .

Proof. Let  $A \in PO(X)$ . Then  $A \in G\text{-}S_pO(X)$ , implies that  $A \subseteq \psi(\text{pint}(A))$ . Also  $A$  can be expressed as  $A = A \cap X$ , where  $A \in PO(X)$  and  $\psi(\text{pint}(X)) = \text{pint}(X)$ . Thus  $A$  is a  $G\text{-}S_p^*$ -set. Conversely, Let  $A \in G\text{-}S_pO(X)$  and  $A$  be a  $G\text{-}S_p^*$ -set. Thus  $A \subseteq \psi(\text{pint}(A)) = \psi(\text{pint}(U \cap V))$ , where  $U \in PO(X)$  and  $\psi(\text{pint}(V)) = \text{pint}(V)$ . Now  $A \subseteq U \cap A \subseteq U \cap \psi(\text{pint}(U \cap V)) = U \cap \psi(U \cap \text{pint}(V)) \subseteq U \cap \psi(U) \cap \psi(\text{pint}(V)) = U \cap \text{pint}(V) = \text{pint}(A)$ . Hence  $A \in PO(X)$ .

**Definition 2.5.** A function  $f: (X, \tau, G) \rightarrow (Y, \sigma)$  is  $G\text{-}S_p^*$ -continuous if for each  $V \in PO(Y)$ ,  $f^{-1}(V)$  is a  $G\text{-}S_p^*$ -set in  $(X, \tau, G)$ .

**Theorem 2.15.** Let  $(X, \tau, G)$  be a grill topological space. Then for a function  $f: (X, \tau, G) \rightarrow (Y, \sigma)$ , the following are equivalent:

- (i)  $f$  is precontinuous;
- (ii)  $f$  is  $G\text{-}S_p$ -continuous and  $G\text{-}S_p^*$ -continuous.

Proof. Straightforward.

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